Exercice 1

Partie 1

1)

$$\begin{aligned}
\text{y(n)-y(n-2)} &= 0,04x(n-1)\\
(Zy)(z)-(Zy)(z)z^{-2} &= 0.04z^{-1}(Zx)(z)\\
(1-z^{-2})(Zy)(z) &= 0.04z^{-1}(Zx)(z)\\
(Zy)(z) &= \frac{0.04z^{-1}}{1-z^{-2}}(Zy)(z)\\
(Zy)(z) &= \frac{0.04z}{z^2-1}(Zy)(z)\\
(Zy)(z) &= \frac{0.04z}{(z-1)(z+1)}(Zy)(z)
\end{aligned}$$

2) a) $e(n)$ est un échelon donc

$$(Zx)(z)=\frac{z}{z-1}$$

En remplacant on obtient :

$$(Zy)(z) = \frac{0.04z^2}{(z-1)^2(z+1)}(Zy)(z)$$

b) Calcul des constantes A, B et C :

$$\begin{aligned}
\frac{0.04\,z}{(z-1)^2(z+1)} &= \frac{A}{(z-1)^2}+\frac{B}{(z-1)}+\frac{C}{(z+1)}\\
&= \frac{A(z+1)+B(z-1)(z+1)+C(z-1)^2}{(z-1)^2(z+1)}\\
&= \frac{(B+C)z^2+(A-2C)z+(A-B+C)}{(z-1)^2(z+1)}
\end{aligned}$$

Par identification on obtient :

$$\left\{\begin{array}{l} B+C=0\\ A-2C=0,04\\ A-B+C=0\end{array}\right.\qquad \left\{\begin{array}{l} B=-C\\ -4C=0,04\\ A=B-C\end{array}\right.\qquad \left\{\begin{array}{l} A=0.02\\ B=0.01\\ C=-0.01\end{array}\right.$$

Finallement :

$$\frac{0.04z}{(z-1)^2(z+1)}=\frac{0.02}{(z-1)^2}+\frac{0.01}{z-1}-\frac{0.01}{z+1}$$

c) Calcul de $y(n)$:

$$\begin{aligned}
\frac{(Zy)(z)}{z} &= \frac{0.04z}{(z-1)^2(z+1)}\\
(Zy)(z) &= \frac{0.02z}{(z-1)^2}+\frac{0.01z}{z-1}-\frac{0.01z}{z+1}\\
(Zy)(z) &= \frac{0.02z}{(z-1)^2}+\frac{0.01z}{z-1}-\frac{0.01z}{z-(-1)}\\
y(n) &= 0.02+0.01-0.01(-1)^n\\
y(n) &= 0.02+0.01(1-(-1)^n)
\end{aligned}$$

Partie 2

1) D'après le formulaire on a :

$$\frac{F(P)}{p} \overset{L^{-1}}{\longmapsto} \int_0^t f(u)du$$

2) Calcul de $s(t)$:

$$\begin{aligned}
s(t) &= \int_0^t sin(20u)du \\
s(t) &= \left[\frac{cos(20u)}{20}\right]_0^t \\
s(t) &= \left(-\frac{cos(20t)}{20}) - (-\frac{1}{20})\right) \\
s(t) &= \left(\frac{1 - cos(20t)}{20}\right)
\end{aligned}$$

d)

$$\begin{aligned}
y(2k) &= 0.02(2k) + 0.01\big(1 - (-1)^{2k}\big) \\
y(2k) &= 0.04k + 0.01(1 - 1) \\
y(2k) &= 0.04k
\end{aligned}$$

$$\begin{aligned}
y(2k+1) &= 0.02(2k+1) + 0.01\big(1 - (-1)^{2k+1}\big) \\
y(2k+1) &= 0.04k + 0.02 + 0.01(1 + 1) \\
y(2k+1) &= 0.04k + 0.04
\end{aligned}$$

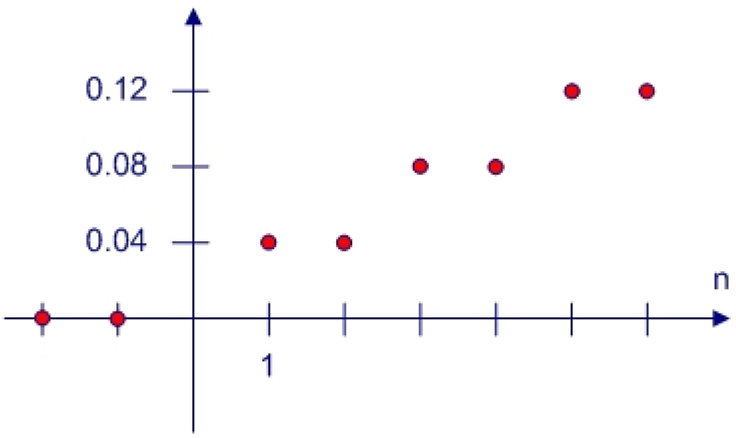
e)

$$\begin{aligned}
y(2k+2) &= 0.02(2k+2) + 0.01\big(1 - (-1)^{2k+2}\big) \\
y(2k+2) &= 0.04k + 0.04 + 0.01(1 - 1) \\
y(2k+2) &= 0.04k + 0.04
\end{aligned}$$

On remarque ainsi que :

$$y(2k+1) = y(2k+2)$$

f) Représentation du signal causal :



Exercice 2

Partie A

1. Calcul de a_0

$$\begin{aligned}
a_0 &= \frac{1}{1}\int_0^1 (\alpha t+\beta)dt \\
a_0 &= \left[\frac{\alpha t^2}{2}+\beta t\right]_0^1 \\
a_0 &= (\frac{\alpha}{2}+\beta)-(0) \\
a_0 &= \frac{\alpha}{2}+\beta
\end{aligned}$$

2. Calcul de b_n

$$bn = 2\int_0^1 (\alpha t+\beta)sin(2n\pi t)dt$$

$$\left|\begin{array}{l} u=\alpha t+\beta \\ dv = sin(2n\pi t)dt \end{array}\right. \qquad \left|\begin{array}{l} du=\alpha dt \\ v=-\dfrac{cos(2n\pi t)dt}{2n\pi} \end{array}\right.$$

$$\begin{aligned}
bn &= 2\left(\left[-(\alpha t+\beta)\frac{cos(2n\pi t)dt}{2n\pi}\right]_0^1 - 2\int_0^1 (\alpha+\beta)sin(2n\pi t)dt\right) \\
bn &= 2\left(\left(-(\alpha t+\beta)\frac{cos(2n\pi t)dt}{2n\pi}\right)-\left(-(\beta)\frac{cos(0)dt}{2n\pi}\right)+\frac{\alpha}{2n\pi}\left[-(\alpha t+\beta)\frac{sin(2n\pi t)dt}{2n\pi}\right]_0^1\right) \\
bn &= 2\left(\left(-\frac{(\alpha+\beta)}{2n\pi}\right)-\left(-\frac{\beta}{2n\pi}\right)\right) \\
bn &= -\frac{\alpha}{n\pi}
\end{aligned}$$

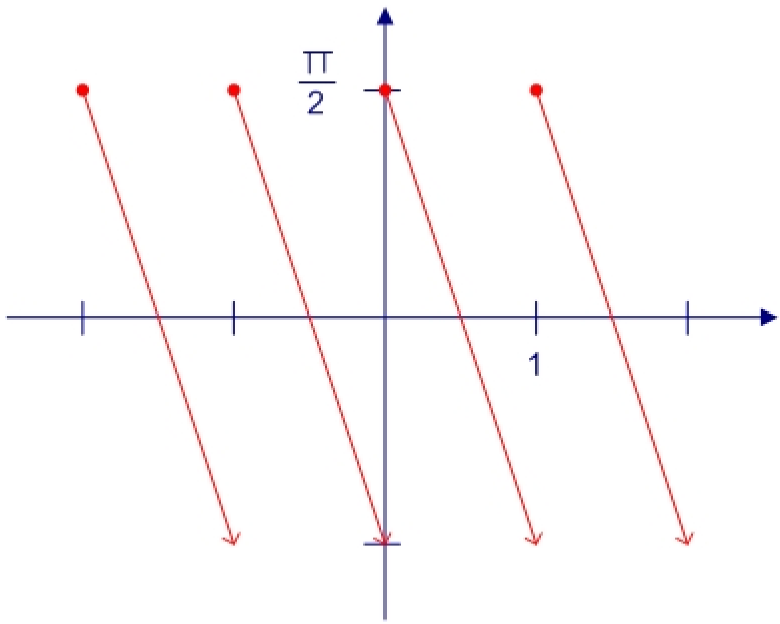
3. a) Déterminer α et β :

$$\left\{\begin{array}{l} \dfrac{\alpha}{2}+\beta=0 \\ -\dfrac{\alpha}{n\pi}=\dfrac{1}{n} \end{array}\right. \qquad \left\{\begin{array}{l} \beta=-\dfrac{\alpha}{2} \\ \alpha=-\dfrac{n\pi}{n} \end{array}\right. \qquad \left\{\begin{array}{l} \beta=\dfrac{\pi}{2} \\ \alpha=-\pi \end{array}\right.$$

On peux maintenant en déduire $f(t)$:

$$f(t) = -\pi t+\frac{\pi}{2}$$

b) Représentation de f(t) :



Partie B

1. Déterminer s_1(t) :

$$s''(t) + s(t) = sin(2\pi t)$$

$$\begin{array}{c|l} 1 & s(t) = Asin(2\pi t) \\ 0 & s'(t) = 2\pi Asin(2\pi t) \\ 1 & s''(t) = -4\pi^2 Asin(2\pi t) \end{array}$$

$$(-4\pi^2 A + A)sin(2\pi t) = sin(2\pi t)$$

Par identification on trouve A :

$$\begin{aligned} -4\pi^2 A + A &= 1 \\ A &= \frac{1}{1-4\pi^2} \end{aligned}$$

$$s''(t) + s(t) = \frac{1}{2}sin(4\pi t)$$

$$\begin{array}{c|l} 1 & s(t) = Bsin(4\pi t) \\ 0 & s'(t) = 4\pi Bsin(4\pi t) \\ 1 & s''(t) = -16\pi^2 Bsin(4\pi t) \end{array}$$

$$(-16\pi^2 B + B)sin(4\pi t) = \frac{1}{2}sin(4\pi t)$$

Par identification on trouve B :

$$\begin{aligned} -16\pi^2 B + B &= \frac{1}{2} \\ B &= \frac{1}{2(1-16\pi^2)} \end{aligned}$$

Finalement on trouve :

$$\begin{aligned} s_1(t) &= Asin(2\pi t) + Bsin(4\pi t) \\ s_1(t) &= \frac{1}{1-4\pi^2} sin(2\pi t) + \frac{1}{2(1-16\pi)^2} sin(4\pi t) \end{aligned}$$

$s''(t) + s(t) = 0 \qquad r^2 + 1 = 0 \qquad$ donc : $\qquad s(t) = C\cos(t) + D\sin(t)$

$$s(t) = C\cos(t) + D\sin(t) + \frac{1}{1-4\pi^2}\sin(2\pi t) + \frac{1}{2(1-16\pi^2)}\sin(4\pi t)$$