BTS électrotechnique 2008

I 1) a)

b) $E(p) = \frac{4}{p}(1-e^{-2p})$

2) $4pS(p) + S(p) = E(p) \Leftrightarrow S(p) = \frac{4(1-e^{-2p})}{p(4p+1)}$

$\Leftrightarrow S(p) = \frac{(1-e^{-2p})}{p(p+1/4)}$ cqfd

3) Par la méthode des pôles il vient $a = 4$ et $b = -4$.

$\Rightarrow \frac{1}{p(p+1/4)} = \frac{4}{p} - \frac{4}{(p+\frac{1}{4})}$

4)

$F(p)$	$1/p$	$\frac{1}{p}e^{-2p}$	$\frac{1}{p+1/4}$	$\frac{e^{-2p}}{p+1/4}$
$f(t)$	$u(t)$	$u(t-2)$	$e^{-t/4}u(t)$	$e^{-\frac{(t-2)}{4}}u(t-2)$

5a) $s(t) = 4(1 - e^{-t/4})u(t) - 4(1 - e^{-(t-2)/4})u(t-2)$

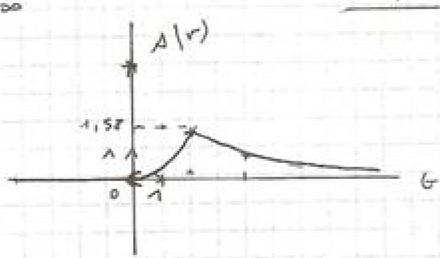
b) $s(t) = \begin{cases} 0 \text{ si } t<0 \\ 4(1-e^{-t/4}) \text{ sur } [0,2[\\ 4(e^{-(t-2)/4} - e^{-t/4}) \text{ sur } [2,+\infty[\\ = 4e^{-t/4}(e^{1/2}-1) \end{cases}$

6) a) $s'(t) = e^{-t/4} > 0$ sur $[0,2[$ donc $s \nearrow$ sur $[0,2[$

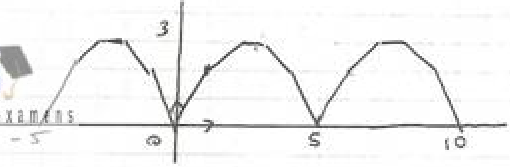
b) $\lim_{t \to 2^-} s(t) = 4 - 4e^{-1/2}$

7 a) $s'(t) = -e^{-t/4}(e^{1/2}-1) < 0$ donc $s \searrow$ sur $[2,+\infty[$

b) $\lim_{t \to +\infty} e^{-t/4} = 0 \Rightarrow \lim_{t \to +\infty} s(t) = 0$



II)



$$f(t) = \begin{cases} 2t & \text{sur } [0,1[\\ t+1 & \text{sur } [1,2[\\ 3 & \text{sur } [2, 5/2[\end{cases}$$

(B) 1) $V_{moy} = \frac{2}{5}\left[\int_0^1 Et\,dt + \int_1^2 (3-E)t + 2E-3\,dt + \int_2^{5/2} 3\,dt\right]$

$= \frac{2}{5}\left[E\left[\frac{t^2}{2}\right]_0^1 + \left[\frac{(3-E)t^2}{2}\right]_1^2 + \left[(2E-3)t\right]_1^2 + 3\times\frac{1}{2}\right]$

$= \frac{2}{5}\times\left[\frac{E}{2} + 2(3-E) - \frac{3}{2} + \frac{E}{2} + 4E - 6 - 2E + 3 + \frac{3}{2}\right] = \underline{\frac{2}{5}(E+3)}$

2) $\underline{b_n = 0 \text{ car } f \text{ paire}}$.

3) a) $\int_0^1 t\cos\left(\frac{2n\pi}{5}t\right)dt$ $\quad u = t \quad u' = 1 \quad v' = \cos\left(\frac{2n\pi}{5}\right)t \quad v = \frac{5}{2n\pi}\sin\left(\frac{2n\pi}{5}\right)t$

$\Leftrightarrow \frac{5}{2n\pi}\left[t\sin\left(\frac{2n\pi}{5}\right)t\right]_0^1 + \frac{25}{4n^2\pi^2}\left[\cos\frac{2n\pi}{5}t\right]_0^1$

$= \frac{5}{2n\pi}\sin\frac{2n\pi}{5} + \frac{25}{4n^2\pi^2}\left(\cos\left(\frac{2n\pi}{5} - 1\right)\right)$ cqfd

$a_n = \frac{4}{5}\int_0^{5/2} f(t)\cos\left(\frac{2\pi n}{5}\right)t\,dt = \underline{\frac{5}{n^2\pi^2}\left[(2E-3)\cos\left(\frac{2n\pi}{5}\right) + (3-E)\cos\frac{4n\pi}{5} - E\right]}$

4) a) $u_5(t) = a_5\cos 2n\pi + b_5\underbrace{\sin 2n\pi}_{0} = a_5 = \frac{5}{25\pi^2}(2E-3+3-E-E) = 0$

donc $\underline{u_5(t) = 0}$

b) $u_3(t) = 0 \Leftrightarrow a_3\cos\frac{6\pi}{5} = 0 \Leftrightarrow a_3 = 0$

$\Leftrightarrow (2E-3)\cos\frac{6\pi}{5} + (3-E)\cos\frac{12\pi}{5} - E = 0$

$\Leftrightarrow -\cos\pi/5 \times 2E + 3\cos\pi/5 + 3\cos\frac{2\pi}{5} - E\cos\frac{2\pi}{5} - E = 0$

$\Leftrightarrow E = \frac{3\cos\pi/5 + 3\cos 2\pi/5}{2\cos\pi/5 + \cos 2\pi/5 + 1} = \underline{1{,}15}.$