Corrigé BTS 2008 Mathématiques groupement A

Repère de l'épreuve : MATGRA1 Session : 2008

Exercice 1 :

PARTIE A

1) a. $S(p) = H(p).E(p)$

$$= \frac{1}{1+2p} \times \frac{1}{p} = \frac{1}{p(1+2p)} = \frac{1}{2p} \times \frac{1}{p+\frac{1}{2}}$$

b. $\frac{\alpha}{p} + \frac{\beta}{p+\frac{1}{2}} = \frac{\alpha p + \frac{\alpha}{2} + \beta p}{p\left(p+\frac{1}{2}\right)}$

Après identification par $\frac{1}{2} \times \frac{1}{p+\frac{1}{2}}$:

$$\begin{cases} \alpha + \beta = 0 \\ \frac{\alpha}{2} = \frac{1}{2} \end{cases} \Leftrightarrow \begin{cases} \alpha = 1 \\ \beta = -1 \end{cases}$$

c. $S(p) = \frac{1}{p} - \frac{1}{p+\frac{1}{2}}$

$$s(t) = \left(1 - e^{-\frac{t}{2}}\right) U(t)$$

2) a. $F(z) = H\left(\frac{10z-10}{z+1}\right) = \frac{1}{1+2\left(\frac{10z-10}{z+1}\right)} = \frac{z+1}{z+1+2(10z-10)} = \frac{z+1}{z+1+20z-20}$

$$= \frac{z+1}{21z-19}$$

b. $x(n) = (0.2n)$

$$X(z) = \frac{z}{z-1}$$

c. $Y(z) = F(z).X(z) = \dfrac{z+1}{21z-19} \times \dfrac{z}{z-1} = \dfrac{z}{z-1} - \dfrac{20}{21}\left(\dfrac{z}{z-\dfrac{19}{21}}\right)$

$$= \frac{z}{z-1} - \frac{20z}{21z-19} = \frac{z(21z-19) - 20z(z-1)}{(z-1)(21z-19)} = \frac{21z^2 - 19z - 20z^2 + 20z}{(z-1)(21z-19)}$$

$$= \frac{z^2+z}{(z-1)(21z-19)} = \frac{z(z+1)}{(z-1)(21z-19)}$$

$$Y(z) = \frac{z}{z-1} - \frac{20}{21} \times \frac{z}{z-\dfrac{19}{21}}$$

$$y(n) = \left(1 - \frac{20}{21} \times \left(\frac{19}{21}\right)^n\right) e(n)$$

3)

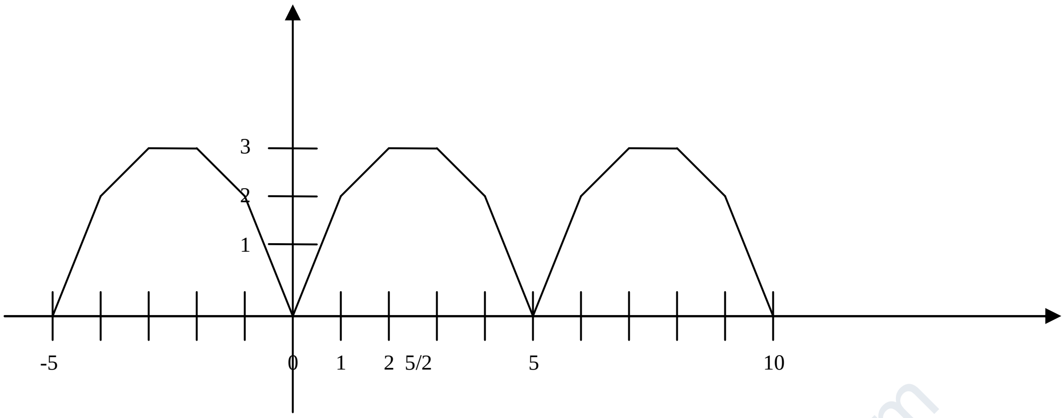
n	y(n)	t = 0,2n	S(t)
0	0,048	0	0
1	0,138	0,2	0,095
5	0,423	1	0,393
10	0,650	2	0,632
15	0,788	3	0,777
20	0,871	4	0,865
25	0,922	5	0,918
50	0,994	10	0,993

Exercice 2

PARTIE A

1) $f(t) = \begin{cases} 2t & sur\ [0;1[\\ t+1 & sur\ [1;2[\\ 3 & sur\ \left[2;\dfrac{1}{2}\right] \end{cases}$

2)



PARTIE B

1) $a_0 = \dfrac{1}{T}\int_0^T f(t)\,d(t) = \dfrac{1}{5}\int_0^5 f(t)\,d(t)$

Fonction paire donc :

$$a_0 = \frac{2}{5}\int_0^{\frac{5}{2}} f(t)\,d(t) = \frac{2}{5}\left(\int_0^1 Et\,d(t) + \int_1^2 (3-E)t + 2E - 3\ d(t) + \int_2^{\frac{5}{2}} 3\,d(t)\right)$$

$$a_0 = \frac{2E}{5}\left(\left[\frac{t^2}{2}\right]_0^1 + \left[\frac{(3-E)t^2}{2} + (2E-3)t\right]_1^2 + [3t]_2^{\frac{5}{2}}\right)$$

$$a_0 = \frac{2E}{5} + \frac{2}{5}\left(2(3-E) + 2(2E-3) - \frac{1}{2}(3-E) - 2E + 3 + 3\left(\frac{5}{2} - 2\right)\right)$$

$$= \frac{2E}{5} + \frac{4}{5}(3-E) + \frac{4}{5}(2E-3) - \frac{1}{2}(3-E) - 2E + 3 + \frac{3}{5} = \frac{1}{5}(2E+3)$$

2) f est une fonction paire donc $b_n = 0, \quad \forall_n \geq 1$

3) a. $\displaystyle\int_0^1 t\times\cos\left(\frac{2n\pi}{5}t\right)d(t)=\left[\frac{5t}{2n\pi}\times\sin\left(\frac{2n\pi}{5}t\right)\right]_0^1-\int_0^1\frac{5}{2n\pi}\sin\left(\frac{2n\pi}{5}t\right)d(t)$

$$=\left[\frac{5t}{2n\pi}\times\sin\left(\frac{2n\pi}{5}t\right)\right]_0^1+\left[\frac{25}{4n^2\pi^2}\cos\left(\frac{2n\pi}{5}t\right)\right]_0^1$$

$$=\left[\frac{5}{2n\pi}\times\sin\left(\frac{2n\pi}{5}\right)-0\right]+\left[\frac{25}{4n^2\pi^2}\cos\left(\frac{2n\pi}{5}\right)-\frac{25}{4n^2\pi^2}\right]$$

$$=\frac{5}{2n\pi}\times\sin\left(\frac{2n\pi}{5}\right)+\frac{25}{4n^2\pi^2}\left(\cos\left(\frac{2n\pi}{5}\right)-1\right)$$

b. $\displaystyle a_n=\frac{2}{T}\int_0^T f(t)\cos(n\omega t)d(t)=\frac{4}{5}\int_0^{\frac{5}{2}} f(t)\cos\left(\frac{2n\pi}{5}t\right)d(t)$

$$=\frac{4}{5}\times\frac{25}{4n^2\pi^2}\left[(2E-3)\cos\left(\frac{2n\pi}{5}\right)+(3-E)\cos\left(\frac{4n\pi}{5}\right)-E\right]$$

$$=\frac{5}{n^2\pi^2}\left[(2E-3)\cos\left(\frac{2n\pi}{5}\right)+(3-E)\cos\left(\frac{4n\pi}{5}\right)-E\right]$$

4) a. $\displaystyle u_5(t)=a_5\cos\left(\frac{2\pi.5}{5}t\right)+b_5$

$$u_5(t)=\frac{5}{25\pi^2}\big((2E-3)\cos(2\pi)+(3-E)\cos(4\pi)-E\big)\cos(2\pi t)$$

$$u_5(t)=\frac{1}{5\pi^2}(2E-3+3-E-E)\cos(2\pi t)=0$$

b. $\displaystyle u_3(t)=0 \quad\Leftrightarrow\quad a_3\cos\left(\frac{6\pi}{5}t\right)=0$

$$\Leftrightarrow\quad \frac{5}{9\pi^2}\left((2E-3)\cos\left(\frac{6\pi}{5}\right)+(3-E)\cos\left(\frac{12\pi}{5}\right)-E\right)\cos\left(\frac{6\pi}{5}t\right)=0$$

$$\Leftrightarrow \quad (2E-3)\cos\left(\frac{6\pi}{5}\right)+(3-E)\cos\left(\frac{12\pi}{5}\right)-E=0$$

$$\Leftrightarrow \quad 2E\cos\left(\frac{6\pi}{5}\right)-3\cos\left(\frac{6\pi}{5}\right)+3\cos\left(\frac{12\pi}{5}\right)-E\cos\left(\frac{12\pi}{5}\right)-E=0$$

$$\Leftrightarrow \quad E\left(2\cos\left(\frac{6\pi}{5}\right)-\cos\left(\frac{12\pi}{5}\right)-1\right)=3\cos\left(\frac{6\pi}{5}\right)-3\cos\left(\frac{12\pi}{5}\right)$$

$$\Leftrightarrow \quad E_0=\frac{3\cos\left(\frac{6\pi}{5}\right)-3\cos\left(\frac{12\pi}{5}\right)}{\left(2\cos\left(\frac{6\pi}{5}\right)-\cos\left(\frac{12\pi}{5}\right)-1\right)}$$

$$E_0 \approx 1{,}15$$